Until now, we
• Defined stress and strain
• Established stress-strain relations for an elastic material
• Learned stress transformation
• Discussed yield (failure) criteria

This week, we will
• Define yield (failure) in metals
• Learn types of stress-strain curves
• Define ductility.
Plastic Deformation

![Graph showing stress vs. strain with regions labeled Elastic energy and Plastic energy.](image)
Typical yield behavior for non-ferrous alloys.

1: True elastic limit
2 (A'): Proportionality limit
3 (A): Elastic limit
4 (B): Offset yield strength
Stress Strain diagram

Point A: Elastic limit. No permanent strain. Depends on instruments.

Point A': Proportional limit. The stress strain curve deviates from linearity.

Point B: Yield strength. The stress which will produce a small amount of permanent deformation, usually 0.002

As the plastic deformation of the specimen increases, the metal becomes stronger (strain hardening).

Eventually, the load reaches a maximum. The maximum load divided by the original area of the specimen is the ultimate tensile strength.
Stress strain diagram for low carbon steel

- Note the critical status for strength specification
  - proportional limit
  - elastic limit
  - yield stress
  - ultimate stress
  - fracture stress

Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Fig. 3–4
Stress Strain diagram comparison

http://www.tanaka-bondingwire.com
Yield Strength: Comparison

Based on data in Table B4, *Callister 7e.*
- **t** = annealed
- **hr** = hot rolled
- **ag** = aged
- **cd** = cold drawn
- **cw** = cold worked
- **qt** = quenched & tempered

**Graphite/Ceramics/Semicond**
- Ti (5Al-2.5Sn) **ag**
- W (pure)
- Cu (71500) **cw**
- Mo (pure)
- Steel (4140) **a**
- Steel (1020) **cd**
- Al (6061) **ag**
- Steel (1020) **hr**
- Ti (pure) **a**
- Ta (pure)
- Cu (71500) **hr**
- Al (6061) **a**
- Ti (pure)
- Sn (pure)

**Polymers**
- PC
- Nylon 6,6
- PET
- PVC **humid**
- PP
- HDPE
- LDPE
- HDPE **dry**
- PC
- PET
- PVC
- PP
- HDPE
- LDPE

**Metals/Alloys**
- Steel (4140) **qt**
- Steel (4140) **a**
- Ti (pure)
- Al (6061) **a**
- Sn (pure)

**Composites/fibers**
- Hard to measure, since in tension, fracture usually occurs before yield.
- Hard to measure, since in ceramic matrix and epoxy matrix composites, since in tension, fracture usually occurs before yield.

Room *T* values

Hard to measure, since in tension, fracture usually occurs before yield.
Material Properties

- Modulus of elasticity (Hooke’s Law)

$$\sigma = E \varepsilon$$

- Modulus of Resilience

$$u_r = \frac{1}{2} \sigma_{pl} \varepsilon_{pl} = \frac{1}{2} \frac{\sigma_{pl}^2}{E}$$

- Modulus of Toughness
  - It measures the entire area under the stress-strain diagram
Plastic Deformation

- Plastic deformation is a non reversible process where Hooke’s law is no longer valid.

- One aspect of plasticity in the viewpoint of structural design is that it is concerned with predicting the maximum load, which can be applied to a body without causing excessive yielding.

- Another aspect of plasticity is about the plastic forming of metals where large plastic deformation is required to change metals into desired shapes.

Elastic and plastic deformation in uniaxial tension
The Flow Curve

• When the load is released the total strain will immediately decrease from $\varepsilon_1$ to $\varepsilon_2$ by an amount $\sigma/E$ (recoverable elastic strain).

• Time dependent elastik behaviour from $\varepsilon_2$ to $\varepsilon_3$ (called anelastik behaviour) is neglected.

• The hysteresis behaviour resulting from unloading and loading from a plastic strain is generally neglected.

• The yield stress on reloading in opposite direction is lower ($\sigma_b<\sigma_a$), called Bauschinger effect. Generally neglected.

[Stress-strain curves for Fe-0.003% C alloy wire]
Stress Strain Curves

completely elastic:

\[ \sigma = E \varepsilon \]

Elastic-Homogeneous Plastic:

\[ \sigma = E \varepsilon \]

\[ \sigma = K \varepsilon^n \]

heterogeneous plastic flow:

If HCP metals deform by slip and twinning (occurs in bursts).
Type IV stress-strain behavior exhibiting a narrow heterogeneous deformation region between initial elastic and final homogenous flow regions. On-set of local yielding occurs at upper yield point A with corresponding load drop to B defined as the lower yield point. After passage of Lüders band throughout the gage section homogenous deformation commences at C.
Deformation of Typical Polymers

Type V stress-strain behavior usually found in crystalline polymers. Minimum in curve reflects cold working and competition between breakdown of initial structure and its subsequent reorganization into a highly oriented strong material.
Example: The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3–19. If a specimen of this material is stressed to 600 MPa, determine the permanent strain that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.
Solution

- When the specimen is subjected to the load, the strain is approximately 0.023 mm/mm.

- The slope of line $OA$ is the modulus of elasticity,

$$E = \frac{450}{0.006} = 75.0 \text{ GPa}$$

- From triangle $CBD$,

$$E = \frac{BD}{CD} = \frac{600 \times 10^6}{CD} = 75.0 \times 10^9$$

$\Rightarrow CD = 0.008 \text{ mm/mm}$
Solution

- This strain represents the amount of *recovered elastic strain*.

- The permanent strain is

$$\varepsilon_{OC} = 0.023 - 0.008 = 0.0150 \text{ mm/mm} \quad (\text{Ans})$$

- Computing the modulus of resilience,

$$\left(u_r\right)_{\text{initial}} = \frac{1}{2} \sigma_{pl} \varepsilon_{pl} = \frac{1}{2} (450)(0.006) = 1.35 \text{ MJ/m}^3 \quad (\text{Ans})$$

$$\left(u_r\right)_{\text{final}} = \frac{1}{2} \sigma_{pl} \varepsilon_{pl} = \frac{1}{2} (600)(0.008) = 2.40 \text{ MJ/m}^3 \quad (\text{Ans})$$

- Note that the SI system of units is measured in joules, where $1 \text{ J} = 1 \text{ N} \cdot \text{m}$.
Idealized Flow Curves

- Rigid - ideal plastic
- Elastic - ideal plastic
- Piecewise linear
Tension Test

The various engineering and true stress-strain properties obtainable from a tension test

Note that:
– the engineering fracture strain \( \varepsilon_f \) and the % elongation are only different ways of stating the same quantity. Also, the %RA and \( \varepsilon_f \) can be calculated from each other.
Measures of Ductility

Ductility is a qualitative, subjective property of a material. It usually indicates the extent to which a metal can be deformed without fracture.

Two methods one can obtain ductility from tension test are:

• the engineering strain at fracture, $\varepsilon_f$, known as elongation

• the reduction in area (R.A.) at fracture, $q$

$$\varepsilon_f = \frac{L_f - L_0}{L_0}$$

$$q = \frac{A_0 - A_f}{A_0}$$

The two properties are obtained by putting the fractured specimen back together, and taking measurements of $L_f$ and $A_f$.

Both elongation and reduction of area are usually expressed as a percentage. The value of $\varepsilon_f$ will depend on the gage length $L_0$ in necked specimens. The reduction in area is a better method of reporting elongation, especially for ductile materials.
Measures of Ductility

• Plastic tensile strain at failure:

\[
% \, EL = \frac{L_f - L_o}{L_o} \times 100
\]

• Another ductility measure:

\[
% RA = \frac{A_o - A_f}{A_o} \times 100
\]
Elongation depends on the original gauge length $L_o$. % elongation decreases as $L_o$ increases.

% Elongation is chiefly influenced by uniform elongation, which is dependent on the strain-hardening capacity of the material.

Reduction of area is more a measure of the deformation required to produce failure and its chief contribution results from the necking process.

Because of the complicated state of stress state in the neck, values of reduction of area are dependent on specimen geometry, and deformation behaviour.

% Elongation and Reduction of area should not be taken as true material properties.
Table 6.2  Typical Mechanical Properties of Several Metals and Alloys in an Annealed State

<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Yield Strength MPa (ksi)</th>
<th>Tensile Strength MPa (ksi)</th>
<th>Ductility, %EL [in 50 mm (2 in.)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>35 (5)</td>
<td>90 (13)</td>
<td>40</td>
</tr>
<tr>
<td>Copper</td>
<td>69 (10)</td>
<td>200 (29)</td>
<td>45</td>
</tr>
<tr>
<td>Brass (70Cu–30Zn)</td>
<td>75 (11)</td>
<td>300 (44)</td>
<td>68</td>
</tr>
<tr>
<td>Iron</td>
<td>130 (19)</td>
<td>262 (38)</td>
<td>45</td>
</tr>
<tr>
<td>Nickel</td>
<td>138 (20)</td>
<td>480 (70)</td>
<td>40</td>
</tr>
<tr>
<td>Steel (1020)</td>
<td>180 (26)</td>
<td>380 (55)</td>
<td>25</td>
</tr>
<tr>
<td>Titanium</td>
<td>450 (65)</td>
<td>520 (75)</td>
<td>25</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>565 (82)</td>
<td>655 (95)</td>
<td>35</td>
</tr>
<tr>
<td>Material</td>
<td>Treatment</td>
<td>Yield Strength (MPa)</td>
<td>Tensile Strength (MPa)</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------</td>
<td>----------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td><strong>Steel Alloys</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1015</td>
<td>As-rolled</td>
<td>315</td>
<td>420</td>
</tr>
<tr>
<td>1050</td>
<td></td>
<td>415</td>
<td>725</td>
</tr>
<tr>
<td>1080</td>
<td></td>
<td>585</td>
<td>965</td>
</tr>
<tr>
<td>1340</td>
<td>Q + T (205°C)</td>
<td>1590</td>
<td>1810</td>
</tr>
<tr>
<td>1340</td>
<td>(425°C)</td>
<td>1150</td>
<td>1260</td>
</tr>
<tr>
<td>1340</td>
<td>(650°C)</td>
<td>620</td>
<td>800</td>
</tr>
<tr>
<td>4340</td>
<td>(205°C)</td>
<td>1675</td>
<td>1875</td>
</tr>
<tr>
<td>4340</td>
<td>(425°C)</td>
<td>1365</td>
<td>1470</td>
</tr>
<tr>
<td>4340</td>
<td>(650°C)</td>
<td>855</td>
<td>965</td>
</tr>
<tr>
<td>301</td>
<td>Annealed plate</td>
<td>275</td>
<td>725</td>
</tr>
<tr>
<td>304</td>
<td></td>
<td>240</td>
<td>565</td>
</tr>
<tr>
<td>310</td>
<td></td>
<td>310</td>
<td>655</td>
</tr>
<tr>
<td>316</td>
<td></td>
<td>250</td>
<td>565</td>
</tr>
<tr>
<td>403</td>
<td>Annealed bar</td>
<td>275</td>
<td>515</td>
</tr>
<tr>
<td>410</td>
<td></td>
<td>275</td>
<td>515</td>
</tr>
<tr>
<td>431</td>
<td></td>
<td>655</td>
<td>860</td>
</tr>
<tr>
<td>AFC-77</td>
<td>Variable</td>
<td>560–1605</td>
<td>835–2140</td>
</tr>
<tr>
<td>PH 15-7Mo</td>
<td></td>
<td>380–1450</td>
<td>895–1515</td>
</tr>
<tr>
<td><strong>Titanium Alloys</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti-5Al-2.5Sn</td>
<td>Annealed</td>
<td>805</td>
<td>860</td>
</tr>
<tr>
<td>Ti-8Al-1Mo-1V</td>
<td>Duplex annealed</td>
<td>950</td>
<td>1000</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>Annealed</td>
<td>925</td>
<td>995</td>
</tr>
<tr>
<td>Ti-13V-11Cr-3Al</td>
<td>Solution + age</td>
<td>1205</td>
<td>1275</td>
</tr>
<tr>
<td><strong>Magnesium Alloys</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AZ31B</td>
<td>Annealed</td>
<td>103–125</td>
<td>220</td>
</tr>
<tr>
<td>AZ80A</td>
<td>Extruded bar</td>
<td>185–195</td>
<td>290–295</td>
</tr>
<tr>
<td>ZK60A</td>
<td>Artificially aged</td>
<td>215–260</td>
<td>295–315</td>
</tr>
<tr>
<td><strong>Aluminum Alloys</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2219</td>
<td>-T31, T351</td>
<td>250</td>
<td>360</td>
</tr>
<tr>
<td>2024</td>
<td>-T3</td>
<td>345</td>
<td>485</td>
</tr>
<tr>
<td>2024</td>
<td>-T6, T651</td>
<td>395</td>
<td>475</td>
</tr>
<tr>
<td>2014</td>
<td>-T6, T651</td>
<td>415</td>
<td>485</td>
</tr>
<tr>
<td>6061</td>
<td>-T4, T451</td>
<td>145</td>
<td>240</td>
</tr>
<tr>
<td>7049</td>
<td>-T73</td>
<td>475</td>
<td>530</td>
</tr>
<tr>
<td>7075</td>
<td>-T6</td>
<td>505</td>
<td>570</td>
</tr>
<tr>
<td>7075</td>
<td>-T73</td>
<td>415</td>
<td>505</td>
</tr>
<tr>
<td>7178</td>
<td>-T6</td>
<td>540</td>
<td>605</td>
</tr>
<tr>
<td><strong>Plastics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABS</td>
<td>Medium impact</td>
<td>—</td>
<td>46</td>
</tr>
<tr>
<td>Acetal</td>
<td>Homopolymer</td>
<td>—</td>
<td>69</td>
</tr>
</tbody>
</table>
True Stress - True Strain Curve

• The annealed structure is ductile, but has low yield stress. Beyond the ultimate tensile stress, plastic deformation becomes localized (called necking), and the engineering stress drop because of the localized reduction in cross-sectional area.

• Engineering stress-strain curve does not indicate that ductile materials continue to strain-harden up to fracture.

• Engineering stress-strain curve does not give the true nature of the deformation of a metal because it is based on the original dimensions of the specimen.

• However, the true stress continues to rise because the crosssectional area decreases and the material work-hardens in the neck region.

• The true-stress-true-strain curves are obtained by converting the tensile stress and its corresponding strain into true values and extending the curve.

• An assessment of the true stress-true strain curve provides a realistic characteristic of the material.
True Strain and Reduction of Area

- According to the concept of unit linear strain 
  \[ \varepsilon = \frac{\delta}{L_0} = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0} \]

- This satisfies for elastic strain where \( \Delta L \) is very small, but not for plastic strain.

- Definition: true strain or natural strain is the change in length referred to the instantaneous gauge length.

\[
d\varepsilon = \frac{dL}{L} \quad \Rightarrow \int d\varepsilon = \int \frac{dL}{L} \quad \therefore \varepsilon_T = \ln\left(\frac{L}{L_0}\right)
\]

- Hence the relationship between the true strain and the engineering strain becomes

\[ \varepsilon_T = \ln(1 + \varepsilon) \]

Also, the volume remains constant in plastic deformation:

\[
\frac{A_0}{A} = \frac{L}{L_0} \quad \varepsilon_T = \ln \frac{L}{L_0} = \ln \frac{A_0}{A}
\]

We can also show that

\[ \sigma_T = \sigma (1 + \varepsilon) \]
**Example:** A tensile specimen with a 12 mm initial diameter and 50 mm gauge length reaches maximum load at 90 kN and fractures at 70 kN. The maximum diameter at fracture is 10 mm. Determine engineering stress at maximum load (the ultimate tensile strength), true fracture stress, true strain at fracture and engineering strain at fracture.

Engineering stress at maximum load

\[
\frac{P_{\text{max}}}{A_{\text{max}}} = \frac{90 \times 10^3}{\pi (12 \times 10^{-3})^2/4} = \frac{90 \times 10^3}{113 \times 10^{-6}} = 796 \text{ MPa}
\]

True fracture stress

\[
\frac{P_f}{A_f} = \frac{70 \times 10^3}{\pi (10 \times 10^{-3})^2/4} = \frac{70 \times 10^3}{78 \times 10^{-6}} = 891 \text{ MPa}
\]

True strain at fracture

\[
\varepsilon_f = \ln \frac{A_0}{A_f} = \ln \left(\frac{12}{10}\right)^2 = 2 \ln 1.2 = 2(0.182) = 0.365
\]

Engineering strain at fracture

\[
\varepsilon_T = \ln (1 + \varepsilon) \quad \exp(\varepsilon_T) = 1 + \varepsilon
\]

\[
\exp(0.365) = 1 + \varepsilon_f \quad \varepsilon_f = 1.44 - 1.00 = 0.44
\]
True stress calculation accounts for change in the cross-section of the sample.

Necking begins at point \( M \) on engineering stress-strain curve and on \( M' \) on true stress – strain curve.

The corrected line accounts for the complex state of stress within the neck region.
True Stress and True Strain

For many metals, the region of uniform plastic deformation of the $\sigma_T - \varepsilon_T$ curve can be approximated by the Ludwik-Holloman power law relationship:

$$\sigma = K \varepsilon^n$$

$\sigma$ = true stress

$\varepsilon$ = true plastic strain

$n$ = strain-hardening coefficient

$K$ = material constant, defined as the true stress at a true strain of 1.0

<table>
<thead>
<tr>
<th>Material</th>
<th>$n$</th>
<th>$K$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-carbon steel (annealed)</td>
<td>0.21</td>
<td>600</td>
</tr>
<tr>
<td>4340 steel alloy (tempered @ 315°C)</td>
<td>0.12</td>
<td>2650</td>
</tr>
<tr>
<td>304 stainless steel (annealed)</td>
<td>0.44</td>
<td>1400</td>
</tr>
<tr>
<td>Copper (annealed)</td>
<td>0.44</td>
<td>530</td>
</tr>
<tr>
<td>Naval brass (annealed)</td>
<td>0.21</td>
<td>585</td>
</tr>
<tr>
<td>2024 aluminum alloy (heat treated—T3)</td>
<td>0.17</td>
<td>780</td>
</tr>
<tr>
<td>AZ-31B magnesium alloy (annealed)</td>
<td>0.16</td>
<td>450</td>
</tr>
</tbody>
</table>

Sensitive to thermomechanical treatment

Annealed specimens: Larger $n$

Cold worked: Smaller
True stress-strain and engineering stress-strain curves for AISI 4140 hot-rolled steel
Hardness

In general, hardness implies a resistance to deformation

For metals, it is a measure of their resistance to permanent or plastic deformation

Commonly measured by resistance to indentation → indentation hardness

Scratch hardness is also used → Moh’s scale

Hardness tests are simple to make, and they can be made on production parts as quality control checks without destroying the part.

The disadvantage is that although hardness of a material depends on the plastic properties, the stress–strain relation cannot be obtained.
# Hardness Testing Techniques

<table>
<thead>
<tr>
<th>Test</th>
<th>Indenter</th>
<th>Shape of Indentation</th>
<th>Load</th>
<th>Formula for Hardness Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinell</td>
<td>10-mm sphere of steel or tungsten carbide</td>
<td><img src="brinell.png" alt="Diagram" /></td>
<td>$P$</td>
<td>$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$</td>
</tr>
<tr>
<td>Vickers</td>
<td>Diamond pyramid</td>
<td><img src="vickers.png" alt="Diagram" /></td>
<td>$P$</td>
<td>$HV = 1.854P/d_1^2$</td>
</tr>
<tr>
<td>Knoop</td>
<td>Diamond pyramid</td>
<td><img src="knoop.png" alt="Diagram" /></td>
<td>$P$</td>
<td>$HK = 14.2P/l^2$</td>
</tr>
<tr>
<td>Rockwell</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Diamond cone</td>
<td><img src="rockwell_a.png" alt="Diagram" /></td>
<td>60 kg</td>
<td>$R_A = 100 - 500t$</td>
</tr>
<tr>
<td>B</td>
<td>0.1-in.-diameter steel sphere</td>
<td><img src="rockwell_b.png" alt="Diagram" /></td>
<td>100 kg</td>
<td>$R_B = 130 - 500t$</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>150 kg</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td>100 kg</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td>60 kg</td>
<td>$R_F = \text{variable}$</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
<td>150 kg</td>
<td>$R_G = \text{variable}$</td>
</tr>
</tbody>
</table>
| E            |                                  |                      | 100 kg | $R_E = \text{variable}$ }
Conversion

Approximate relation between several hardness scales.

Relation between Vickers and Moh’s hardness scales.
Conversion

Both tensile strength and hardness are indicators of a metal’s resistance to plastic deformation → they are roughly proportional.

For heat treated plain carbon steel and medium alloy steels

\[ S_{UTS} \text{ (MPa)} = 3.45 \text{ (BHN)} \]