2. MODELING and SIMULATION

Direct (forward) solution of an EMI scattering problem provides
- good insight for better system design,
- signatures for all object/sensor orientations,
- intelligent inversion strategies.

These are experimentally very difficult to obtain. Many numerical and analytical methods are used for this aim. In order to calculate the scattered fields, the primary (incident) field can be subtracted from the total field at the half-space \cite{Gao2000}. The background response is not the same at all frequencies.

Electromagnetic field propagates at a speed of light in air (lossless). After passing into the soil (earth), it slows down considerably and becomes almost vertical means parallel to the soil surface. Due to the soil loss, its amplitude decays (a highly dispersive plane wave) as well as phase rotation \cite{Won1998}. According to wavelength, vertical scattering mechanism is shown below.

\[ \lambda \]
\[ h \]
\[ \lambda >> h \]
\[ \lambda \approx h \]
\[ \lambda << h \]

Vertical scattering at low frequencies due to long wavelength.

Solution methods (analytical or numerical) for investigation of a multi-frequency (sweep-frequency) EMI system require that the obtained solutions must be valid for all (any) swept frequencies ranges. Thus, the solution in the considered frequency band can be obtained by repeating the solution for every discrete frequency which corresponds to the considered frequency band\footnote{In experimental side, synchronization concept of the sources can be used for the subtraction mentality. This can be inherently performed by phase lock analyzer or lock-in amplifier. Alternatively, a two output signal generator or usage of two antennas (in the sense of hardware usage) can also be used for this aim.}. However, this way is computationally expensive. The used number of the discrete frequencies is also questionable for desired resolution based accuracy.

At low frequencies, the loop antennas generate dominant magnetic fields with small electric fields. Therefore, the only magnetic fields are generally considered as a source field. Nevertheless, calculation errors originated from the negligence of electric field are investigated for human body problem. Accordingly, in the case of the very low E/H ratios, the negligence is acceptable while the errors increase with ratio. While the only source H field generates symmetric field distributions of sphere, the source E and H fields generate not symmetric field distributions \cite{Part2013}.

In principal, circular symmetry greatly reduces to the mathematical analysis. The calculation of total field (related to the secondary voltage of the coil) is of low interest because it consists of all information about soil properties, coil structures and metallic target. In order to extract information more about the object, total field (secondary voltage) has to be compensated or balanced. In fact, this is related to the subtraction of incident field(s) \cite{Krueger2011}.

Two fundamentals approaches can be used to model the multi-frequency EMI systems \cite{Won1980}:

\[ \text{compensated or balanced} \]
- Maxwell’s (Wave) Equations based: In this way, generally a magneto quasi-static solution is formulated at low frequencies. In order to satisfy the magneto quasi-static condition, whole problem space (both object and open space) must be considered as a lossy medium. In this case, displacement currents can be neglected \( \frac{\partial \mathbf{D}}{\partial t} \cong 0 \) in Maxwell equations. Thus, the problem is converted to a diffusion problem \( \nabla \times \vec{H} = \vec{J} \) inside and outside the object.

In some cases, induced currents within outside region (air or soil) are also be neglected due to the small electric field and conductivity of the outside region. In this case, the problem in the outside region becomes static (magnetic field is irrotational \( \nabla \times \vec{H} = 0 \)) [Shubitidze et al, 2002b] and can be formulated for a field component as

\[
\Delta u_1 + k_1^2 u_1 = 0 \quad ; \quad \text{inside}
\]
\[
\Delta u_2 = 0 \quad ; \quad \text{outside}
\]

where \( u_1 \) and \( u_2 \) are the inside and outside fields, respectively. Under the magneto quasi-static assumption, the wavenumber is \( k_1 = \sqrt{j\omega \sigma \mu} \). The unknown fields are calculated by using boundary and continuity conditions such as matching the tangential field components of the vectors. Anyway, it is necessary to remember that both solutions are approximate. Therefore, limitations, accuracies and validity range of the quasi-static solution obtained by quasi-static approximation must be studied by comparing the exact solution obtained in terms of spherical harmonics [Sebak et al, 1984].

At low frequencies, analytical solutions are rare and valid for only simple geometries. They are generally band limited when conductivity of the soil is finite (realistic ground). Besides them, present (some) numerical works need more computer storage, time and resolution, especially for 3D modeling. Therefore, the numerical methods cannot be feasible for 3D structures.

- Modal based: The solutions are generally reported for only a frequency or discrete set of frequencies. Principally, frequency translation from one frequency to another is possible using a phase-diagram. However, this will wrong in the case of finite conductive ground [Parasnis, 1971], [Won, 1980].

Ewald and Krüger presented a review about analytical and numerical methods for the modeling of the inductive sensors in a metal detection application. They generally discussed usefulness of the mathematical modeling and especially for estimation of the coil impedance. It is highlighted that the numerical solutions have to be used because the analytical solutions are only available for the simplest cases (objects). The commercial solvers OPERA, MAXWELL, FEMLAB, VIC and MAFIA in which finite difference and finite element methods used are evaluated for the numerical inductive sensor modeling. A metal detector in a rotationally symmetric system and in a planar layered medium is investigated as numerical examples. It is noted that 3D problem must be solved for better modeling. The signal processing and inverse problem methods are also necessary in these problems [Ewald and Krüher, 2005].